Homework 2

(Due date: February 1st @ 5:30 pm)
Presentation and clarity are very important! Show your procedure!

Problem 1 (32 pts)

- In ALL these problems (a, b, c), you MUST show your conversion procedure. **No procedure ≡ zero points.**

a) Convert the following decimal numbers to their 2's complement representations: binary and hexadecimal. (12 pts)
   - ✓ -31.3125, 37.5078125, -256.65625, -391.25.

b) Complete the following table. The decimal numbers are unsigned: (8 pts.)

Decimal	BCD	Binary	Reflective Gray Code
397			
		100111110	
			1011100010
256			
			10011100
		1110010	
			10101001
	001001011001		

c) Complete the following table. Use the fewest number of bits in each case: (12 pts.)

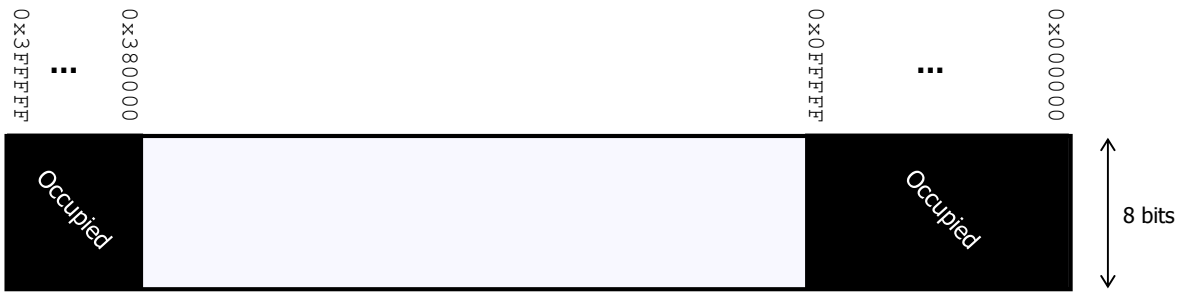
REPRESENTATION			
Decimal	**Sign-and-magnitude**	**1's complement**	**2's complement**
-133			
			100000000
		101100111	
	11010101		
			1001100
		01101001	

Problem 2 (20 pts)

a) What is the minimum number of bits required to represent: (2 pts)
   - ✓ Memory addresses from 0 to 8192?
   - ✓ 32767 symbols?

b) A microprocessor has a 28-bit address line. The size of the memory contents of each address is 8 bits. The memory space is defined as the collection of memory positions the processor can address. (6 pts)
   - What is the address range (lowest to highest, in hexadecimal) of the memory space for this microprocessor? What is the size (in bytes, KB, or MB) of the memory space? 1KB = 2^{10} bytes, 1MB = 2^{20} bytes, 1GB = 2^{30} bytes
   - A memory device is connected to the microprocessor. Based on the size of the memory, the microprocessor has assigned the addresses 0xB1C0000 to 0xB1FFFFF to this memory device. What is the size (in bytes, KB, or MB) of this memory device? What is the minimum number of bits required to represent the addresses only for this memory device?

c) The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. (12 pts)
   - What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor?
   - If we have a memory chip of 512KB, how many bits do we require to address 512KB of memory?
   - We want to connect the 512KB memory chip to the microprocessor. Provide a list of all the possible address ranges that the 512KB memory chip can occupy. You can only use the non-occupied portions of the memory space as shown below.



PROBLEM 3 (38 PTS)

a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits n to represent both operators. Indicate every carry (or borrow) from c_0 to c_n (or b_0 to b_n). For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher byte. (8 pts)

Example (n=8):

✓ 54 + 210

	c_8=1	c_7=1	c_6=1	c_5=1	c_4=0	c_3=1	c_2=1	c_1=0	c_0=0
54 = 0x36 =		0	0	1	1	0	1	1	0 +
210 = 0xD2 =		1	1	0	1	0	0	1	0
Overflow! →	1	0	0	0	0	1	0	0	0

✓ 77 - 194

	Borrow out! → b_8=1	b_7=0	b_6=0	b_5=0	b_4=0	b_3=0	b_2=1	b_1=0	b_0=0
77 = 0x4D =		0	1	0	0	1	1	0	1 -
194 = 0xC2 =		1	1	0	0	0	0	1	0
		0	0	0	0	1	0	1	1

✓ 211 + 99
✓ 101 + 35
✓ 51 - 96
✓ 256 - 57

b) We need to perform the following operations, where numbers are represented in 2's complement: (24 pts)

✓ -77 + 216
✓ -129 + 128
✓ 313 + 711
✓ -62 + 99
✓ -122 - 26
✓ 167 + 512

- For each case:
  - ✓ Determine the minimum number of bits required to represent both summands. You might need to sign-extend one of the summands, since for proper summation, both summands must have the same number of bits.
  - ✓ Perform the binary addition in 2's complement arithmetic. The result must have the same number of bits as the summands.
  - ✓ Determine whether there is overflow by:
    i. Using c_n, c_{n-1} (carries).
    ii. Performing the operation in the decimal system and checking whether the result is within the allowed range for n bits, where n is the minimum number of bits for the summands.
  - ✓ If we want to avoid overflow, what is the minimum number of bits required to represent both the summands and the result?

c) Get the multiplication results of the following numbers that are represented in 2's complement arithmetic with 4 bits. (6 pts)
✓ 0101×0100, 1001×0111, 1011×1101.

PROBLEM 4 (10 PTS)

- Complete the timing diagram (signals DO and $DATA$) of the following circuit. The circuit in the blue box computes the unsigned summation T+6, with the result having 5 bits.
  For example: if T=1001 → DO=1001 + 0110 = 01111. If T=1100 → DO=1101+0110 = 10011.